

IITK-PHY-99-51  
hep-ph/9905576**TRILINEAR R-PARITY VIOLATION: THEORY TO EXPERIMENT***Sreerup Raychaudhuri*

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E-mail: [sreerup@iitk.ac.in](mailto:sreerup@iitk.ac.in)**ABSTRACT**

Supersymmetric models without conservation of  $R$ -parity are reviewed and low-energy constraints on the extra trilinear couplings listed. Current searches at the LEP, Tevatron and HERA colliders are then summed up. Prospects for further study, especially at future colliders, are briefly touched upon.

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**1. INTRODUCTION TO R-PARITY AND R-PARITY VIOLATION**

As we enter a new millenium, supersymmetry (SUSY) appears to be one of the best options for physics beyond the Standard Model (SM). One of the cornerstones of conventional searches for SUSY is the idea of a conserved quantum number called  $R$ -parity, which leads to missing energy-momentum triggers for SUSY signals. We first examine the rationale for conserving  $R$ -parity.

In the minimal supersymmetric extension of the SM, or MSSM, as it is commonly dubbed, to each particle  $P$  in the SM, there corresponds a *superfield*  $\hat{P}$  which contains the field  $P(x)$ , its SUSY partner (sparticle) field  $\tilde{P}(x)$  and an auxiliary field  $F^P(x)$ . The last of these has no dynamics and can be easily integrated out of the action. The simplest prescription for obtaining (most of) the interactions of the matter sector of the MSSM is to take the terms in the SM Lagrangian and replace each field  $P(x)$  by the corresponding superfield  $\hat{P}$ . This leads to the generation of a superpotential, from which the interaction vertices can be obtained by standard rules of composition. This results in the well-known prescription for generating the vertices of the MSSM, namely, to take each SM vertex and replace a *pair* of SM particles by their SUSY partners (sparticles). This prescription is equivalent to postulating[1] conservation of a multiplicative quantum number, namely  $R$ -parity, which is defined by

$$R_p = (-1)^{3B+L+2S} ,$$

where  $B, L, S$  denote, respectively, the baryon number, lepton number and intrinsic spin of the particle. Each SM particle has  $R_p = +1$ , while sparticles generically have  $R_p = -1$ . Conservation of  $R_p$  is not really a surprising feature, since the MSSM interactions constructed by the method described above may be expected to retain the symmetries of the SM, where, as is well-known, the quantum numbers  $B$  and  $L$  are conserved separately. It is natural, therefore, to expect the same to hold for the MSSM. This immediately leads us to conservation of  $R$ -parity.

$R$ -parity conservation, in fact, turns out to be a very convenient feature of SUSY models. For one thing, it means that sparticles are always produced in pairs and hence the *lightest* sparticle (LSP) must be stable. The LSP will actually behave like a heavy neutrino, since it can only interact with ordinary matter by exchanging other (heavy) sparticles. Once a sparticle has been produced in some high energy process, all cascades resulting from its decay must end in production of an LSP, which (like a neutrino) escapes the detectors, generating characteristic missing energy signals. This facilitates experimental detection, especially at hadron colliders, by providing a simple trigger to base the searches on. For this reason, over the years, a considerable literature has grown up around these missing energy signals, and the same ideas have been extensively used in experimental searches for SUSY.

Another good feature of conserved  $R$ -parity is that the LSP turns out to be an excellent candidate for the cold dark matter component of the Universe. This is demanded by theoretical models which close the Universe and especially by theories which postulate an inflationary epoch in the early Universe. Observational evidence, such as the rotation curves of spiral galaxies, does seem to require some dark matter, and the observed level of fluctuations in the microwave background also requires a cold dark matter component in order to explain galaxy formation. However, it is only fair to point out that there could be other candidates for cold dark matter, such as invisible axions, other wimps or machos.

### 1.1 Violation of $R$ -Parity

The story of  $R$ -parity conservation does not end here, however. It is surely not reasonable to impose a dynamical symmetry merely because it has agreeable consequences. We must therefore, ask ourselves if the arguments leading to the postulate of  $R$ -parity conservation are really compelling. It turns out that the weak point of the above scenario is that  $B$  and  $L$  are, in a sense, accidental symmetries of the SM and are not built into the gauge symmetry. Hence, the moment one seeks to extend the SM (*e.g.*, to get a GUT), the possibility of  $B$  and/or  $L$ -violation must be encountered. The MSSM is just one case of this[2]. Here the source of  $L$ -violation lies in the well-known fact that there are *two* Higgs doublets instead of one, as in the SM. This is forced upon us by the requirement of holomorphicity of the superpotential as well as arguments from anomaly cancellation: these arguments also tell us that the two doublets must have opposite hypercharges. Thus, there must be a scalar doublet superfield  $\hat{H}_2$  with hypercharge  $Y = -1$  in addition to the SM-like  $\hat{H}_1$ , which has  $Y = +1$ . It is obvious that the  $\hat{H}_2$  has gauge quantum numbers identical to those of the left-handed lepton superfields  $\hat{L}_i$  and hence can *mix* with all of them. Accordingly, to the  $R$ -parity conserving superpotential of the MSSM, we can always add terms with the  $\hat{H}_2$  replaced by a generic lepton superfield  $\hat{L}_i$  and some unknown coupling(s),

$$\begin{aligned} \mathcal{W} = & +\mu\hat{H}_1\hat{H}_2 + h_{ik}^L\hat{L}_i\hat{H}_2\hat{E}_k^c + h_{ik}^D\hat{Q}_i\hat{H}_2\hat{D}_k^c + \dots \\ & +\kappa_i\hat{H}_1\hat{L}_i + \lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k^c + \lambda'_{ijk}\hat{Q}_i\hat{L}_j\hat{D}_k^c \\ & +\lambda''_{ijk}\hat{U}_i^c\hat{D}_j^c\hat{D}_k^c . \end{aligned} \tag{1}$$

It is also possible to violate baryon number by multiplying three right-handed  $SU(2)_L$ -singlet quark superfields together and this gives the last term in Eq. (1). In the above equation, the indices  $i, j, k$  represent generations of SM fermions. The condition that the

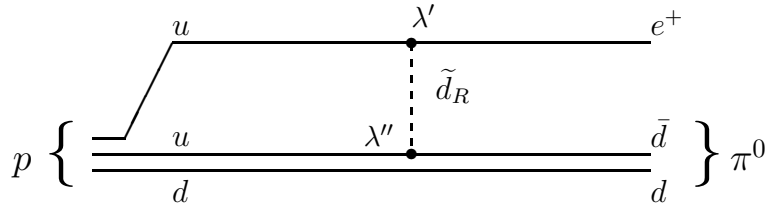
superpotential be holomorphic in the fields requires that the  $SU(2)_L$  doublets be multiplied using the  $SU(2)$  product  $\epsilon_{ab}\Phi_1^a\Phi_2^b$  rather than the simpler  $\Phi_1^\dagger\Phi_2$ . This ensures that the couplings  $\lambda_{ijk}$  are antisymmetric in  $i, j$ . Similarly since the quark superfields belong to  $\mathbf{3}$  of  $SU(3)_c$ , the term in the Lagrangian must be the singlet in the decomposition of  $\mathbf{3}\times\mathbf{3}\times\mathbf{3}$ , *i.e.* totally antisymmetric in colour indices. This ensures that the  $\lambda'_{ijk}$  are also antisymmetric in the flavour indices  $j, k$ .

At first sight it seems as if the bilinear terms  $\kappa_i\hat{H}_1\hat{L}_i$  can be rotated away leaving only the trilinear ones. This is, in fact, stated in several (early) works on the subject. However, this rotation would mean that the Higgs superfield  $\hat{H}_2$  acquires an admixture of leptonic superfields and this will show up in the scalar potential of the theory. It is then possible for the sneutrino<sup>1</sup> to acquire a vacuum expectation value (VEV), leading to spontaneous violation of  $R$ -parity. In such scenarios, the charginos mix with the charged leptons while the neutralinos mix with the neutrinos. Similar mixings occur among the scalars of the theory. Even if the sneutrino does not develop a VEV, it is possible to have bilinear  $R$ -parity violation on the same footing as trilinear  $R$ -parity violation. Moreover, bilinear terms induce the trilinear terms (with a definite structure) through the Higgs Yukawa couplings and hence, in a sense, such models are more predictive than the ones discussed in this article. There exists a rich literature[3] on the subject of bilinear and spontaneous  $R$ -parity violation which is well worth perusal. Phenomenological aspects of these scenarios require a detailed consideration, but we shall not attempt to discuss them in this article.

It is also worth mentioning that the above interactions tell us that in the presence of  $\lambda$  couplings, the sleptons behave as *dileptons*; in the presence of  $\lambda'$  couplings, the squarks behave as *leptoquarks*; in the presence of  $\lambda''$  couplings, the squarks behave as *diquarks*. Such objects at the electroweak scale have been predicted in composite models, but here it is elementary particles which exhibit similar behaviour. Thus any search for scalar dileptons, diquarks or leptoquarks becomes automatically applicable to  $R$ -parity-violating effects. Vector excitations of this nature cannot be mapped to any SUSY effects and, therefore, are irrelevant for our purposes.

## 1.2 Proton Decay

One of the immediate consequences of simultaneous violation of  $B$  and  $L$  is known to be fast proton decay. Most Grand Unified Theories (GUT's) which try to unify the strong and electromagnetic interactions within a single (broken) gauge symmetry do predict proton decay as a consequence. In the MSSM too, if  $R$ -parity is violated in a maximal sense, *i.e.*, all the extra couplings in Eq.(1) are present, one can have proton decay through diagrams such as the one in Figure 1.



**Figure 1.** Typical diagram for proton decay  $p \rightarrow e^+\pi^0$  through  $\lambda'$  and  $\lambda''$  couplings.

<sup>1</sup>like the neutral Higgs boson, with which, indeed, it mixes

The amplitude for this process can immediately be estimated as

$$\mathcal{A}(p \rightarrow \ell^+ \pi^0) \sim \frac{\lambda' \lambda''}{M_{d_R}^2} . \quad (2)$$

Of course, this is not the only possible diagram, but all others have similar amplitudes. Unless one makes very special choices of phase (which would then have to be explained), it is unlikely that there will be large cancellations between different (coherent) diagrams and hence, one can make reasonable estimates of bounds using the above result.

Till date all experimental searches for proton decay have yielded negative results, leading to a lower bound on the proton lifetime of  $\sim 10^{32}$  years: this immediately constrains[4] the product  $\lambda' \lambda''$  in the above equation to be  $\sim 10^{25}$  or smaller, for  $M_{d_R}^2 <$  a few TeV. Rather than take on the burden of explaining such an unnaturally small number, it is more reasonable to assume that the product  $\lambda' \lambda''$  vanishes *identically*. Assuming this to be the case, if we wish this result to hold at all scales, it is clear that we must postulate some symmetry to protect it.  $R$ -parity is just such a symmetry, since it forbids both the  $\lambda'$  and the  $\lambda''$  terms, but it is something of an overkill, since it is enough to have either of the factors vanish *i.e.* we can either have  $L$  conserved, with vanishing of  $\lambda'$ , or have  $B$  conserved, with vanishing  $\lambda''$ . In fact, since the early work of Ref. [4], several papers[5] have attempted to put bounds on all possible products of  $R$ -parity-violating couplings which appear in various proton-decay modes.

### 1.3 Discrete Symmetries

In the MSSM,  $R$ -parity is a relic of a global  $U(1)$  symmetry called  $R$ -symmetry, which is broken to  $Z_2$  when SUSY itself is broken. We can ensure conservation of  $L$  by postulating *lepton parity*, another  $Z_2$  symmetry under which all lepton superfields flip sign, while other superfields remain unchanged. Similarly, one can ensure conservation of  $B$  by postulating *baryon parity*, under which all quark superfields change sign, while other superfields remain unchanged. These parities must be imposed by hand in the MSSM[5] — just as  $R$ -parity itself is — and no compelling dynamical motivation has yet been discovered for either.

The imposition of lepton or baryon-parity, however, immediately leads one to ask the question: if the MSSM is the low-energy effective theory of a SUSY GUT, then, at some high scale, the quarks and leptons must belong to the same gauge multiplet. It is difficult, then, to visualise part of the multiplet being even and part being odd under a discrete transformation such as lepton or baryon parity.

Two approaches to this paradox are possible. The (conceptually) simpler one is to assume that grand unification takes place within the framework of a string theory[6] without any unified gauge group. In such models, one does not need to put quarks and leptons in the same multiplet, and hence there is no objection to having them transform differently under a  $Z_2$  symmetry. Even in the context of ordinary grand unification, however, it has been shown[7] that it is possible to add  $R$ -parity conserving (nonrenormalisable) operators to the Lagrangian of the theory; these, after the GUT symmetry has been broken and quarks and leptons acquire separate identities, metamorphose into  $R$ -parity breaking (renormalisable) operators. Thus, in these models,  $R$ -parity violation is possible without running foul of proton decay constraints.

It has also been argued that baryon parity is better motivated theoretically than lepton parity. The reason is that it appears[8] that quantum gravity effects could maximally violate any discrete symmetry of a theory which is not a relic of a gauged symmetry. At a high scale, then, we would require all the discrete symmetries of the Lagrangian to be such relics. An analysis[9] of the possible candidate models shows that the only ones which are compatible with an anomaly-free gauge theory, in the first place, are  $R$ -parity and baryon parity. Accordingly, if we wish to break  $R$ -parity, it seems that  $L$  violation is favoured. However, one should note that none of these arguments is really watertight and depend on various approximations and *ansätze* which could be called into question[10] if necessary.

Finally, one is driven to the obvious question, as to whether  $R$ -parity-violating models are consistent with supersymmetric coupling constant unification. The answer seems to be rather parameter-space dependent[11], but it appears that low energy solutions of the renormalisation group in which  $R$ -parity violating couplings partially drive the evolution are possible. In fact, these have been used to put (somewhat loose) bounds on some of the  $\lambda''$  couplings[12].

## 2. LOW-ENERGY PHENOMENOLOGY

Using the symmetry properties of the  $R$ -parity-violating couplings in Eq.(1) it is possible to count the number of extra parameters introduced into the MSSM. There are 9  $\lambda_{ijk}$ 's, 27  $\lambda'_{ijk}$ 's and 9  $\lambda''_{ijk}$ 's, making a total of 45 new parameters in all. Some of these vanish because of the proton decay constraint — for example, one can have 36  $\lambda_{ijk}$ 's and  $\lambda'_{ijk}$ 's, while the  $\lambda''_{ijk}$ 's vanish; or one can have 9  $\lambda''_{ijk}$ 's, while the rest vanish. Even then, enough free parameters remain to make a phenomenological analysis valueless if *all* of them are taken to be free parameters<sup>2</sup>.

In order to study the phenomenological behaviour of SUSY without  $R$ -parity conservation, then, the usual hypothesis one makes is that just one of the  $R$ -parity-violating couplings is dominant and the others are, for all practical purposes, zero. The common approach is to assume that this is true in the physical basis for fermions. One can justify this by two plausibility arguments: first, this is indeed the pattern of SM Yukawa couplings ( $y_f$ ), where  $y_t$  is much larger than the others; secondly, there exist various phenomenological bounds on *products* of pairs of  $R$ -parity-violating couplings[14] which leads us to suspect that these products might actually vanish. An alternative approach[15] is to assume that one coupling is dominant in the gauge basis (before the electroweak symmetry is broken) and others are generated because of the quark (lepton?) mixing in the physical basis through the Cabibbo-Kobayashi-Maskawa matrix in the quark (lepton?) sector. In the latter case, bounds on products of couplings translate into bounds on the single dominant coupling in the gauge basis. This latter procedure may seem a more rational one, since the former requires a conspiracy among the couplings to have various cancellations in the physical basis, leaving only one of them dominant. However, as some such mechanism is probably at work in the SM making  $y_t$  large in the physical basis, the question is basically one of philosophy. We adopt the attitude that one of the couplings in the physical basis is dominant, leaving the explanation to a deeper theory when that should become available.

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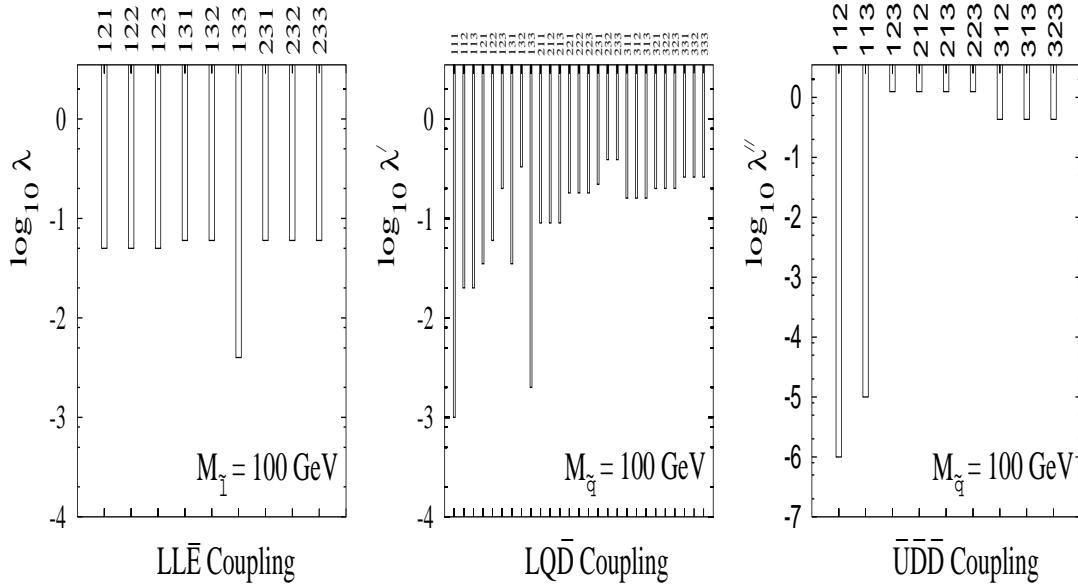
<sup>2</sup>This scenario has, in fact, been described[13] as a ‘nightmare’.

## 2.1 Four-fermion operators

Assuming that one (at a time) out of the 45 possible couplings is dominant, the Lagrangian arising out of Eq.(1) leads[16] to low-energy effective four-fermion operators contributing to the physics of various processes at low and intermediate energy scales. These can be as varied as

- violation of charged current universality,
- tau decays to final states with electrons/muons,
- pion decays to electrons/muons,
- generation of  $\nu_e$  masses,
- $\nu_\mu$  deep inelastic scattering,
- neutrinoless double beta decay,
- atomic parity violation,
- $D^0 - \bar{D}^0$  mixing,
- $D^+$  decays,
- leptonic decays of the  $Z$ , quantified by  $R_\ell = \Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow \ell^+ \ell^-)$ ,
- heavy nucleon decay,
- $n - \bar{n}$  oscillations.

Bounds from these have been reviewed several times in the literature[17, 18, 19] and will not be discussed here. A summary of some of these is depicted in Figure 2. These have been taken from Ref.[17], and some of them are a little outdated: some updates may be found in Refs. [18] and [19].



**Figure 2.** Bounds on R-parity-violating couplings[17] assuming the mass of the exchanged sfermion is 100 GeV. The vertical bars correspond to ruled-out regions.

A glance at the figure will immediately reveal that bounds on couplings involving the first generation are generally the best, weakening as one goes through the second generation to the third generation. Accordingly, one can predict the greatest deviations from the SM in processes involving the third generation. Of course, these effects are

generally harder to measure, which is why the bounds are weak in the first place.

It is also worth pointing out that these bounds are, in a sense, unrealistic in the sense that it is assumed, while deriving them, that there is no other supersymmetric contribution to the process(es) under consideration. This would be justified if the sparticle exchanged in an  $R$ -parity-violating interaction was the only light one, others being heavy enough to decouple. Such an assumption is rather artificial. A complete study would, however, involve all the parameters of the MSSM and render the analysis quite messy. Preliminary investigations[20] show, however, that many of the existing bounds are actually quite robust.

The recent confirmation of an atmospheric neutrino deficit from the experiments conducted by the Super-Kamiokande Collaboration has led to an explosion in the literature of speculations about finite neutrino masses and large-angle mixing.  $R$ -parity violation is one mechanism which can generate these, and, in fact, the rather stringent bound on  $\lambda_{133}$  comes from a consideration of the  $\nu_e$  mass[3]. For further discussions of this extremely interesting aspect of  $R$ -parity violation the reader is referred to the literature[21].

### 3. COLLIDER SIGNALS FOR $R$ -PARITY VIOLATION

In models where  $R$ -parity is conserved, the LSP is a stable particle which escapes detection at colliders and leads, as we have explained above, to large missing energy and momentum signatures. If  $R$ -parity is violated, this is no longer true. The LSP can decay to multi-fermion states through the  $R$ -parity-violating couplings. Since this is the sole decay mode of the LSP, the actual magnitude of the coupling is not very important, so long as it allows the LSP to decay within the detector.

Of greater importance, once we allow  $R$ -parity to be broken, is the *identification* of the LSP. Since it decays and is therefore useless as a dark matter candidate, all the phenomenological arguments in favour of its being a sneutrino or neutralino are no longer valid. In fact, in the most general  $R$ -parity-violating scenario, *any* of the sparticles can be the LSP. One would, in general, have to construct separate search strategies for each case. It is usual, therefore, to invoke a specific pattern of SUSY-breaking in order to predict the mass spectrum — this enables us to predict which particle is the LSP. A popular choice of model is the *constrained* MSSM, or cMSSM, which assumes a common scalar mass ( $m_0$ ), a common gaugino mass ( $M_{1/2}$ ) and common trilinear couplings ( $A$ ) at some high scale, usually identified with the GUT scale. In this scenario, one possibility — valid over a large part of the MSSM parameter space — is for the lightest neutralino to be the LSP, just as it is in the case when  $R$ -parity is conserved. Another possibility is for the LSP to be one of the sleptons. Most phenomenological studies and experimental searches concentrate — for obvious reasons — on the former scenario. The latter scenario(s) has not been studied in similar detail, though they have been considered<sup>3</sup> in specific contexts.

A slepton LSP can decay directly into two leptons or two quarks if  $L$  is violated; if  $B$  is violated, it will have four-body decays into a lepton and three quarks. On the other hand, a neutralino LSP will always have three-body decays of the form[24]

$$\tilde{\chi}_1^0 \xrightarrow{\lambda} \ell_1^+ \ell_2^- + \cancel{E}_T, \quad \tilde{\chi}_1^0 \xrightarrow{\lambda'} \ell^\pm + 2 \text{ jets}, \quad \tilde{\chi}_1^0 \xrightarrow{\lambda''} 3 \text{ jets}, \quad (3)$$

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<sup>3</sup>An example of this is the explanation[22] of the excess in four-jet events claimed by the ALEPH Collaboration in 1995. Some explanations of the excess in high- $Q^2$  events seen at HERA in 1997 required a charmed squark to be the LSP[23].

and these will lead to rather distinctive final states at colliders.

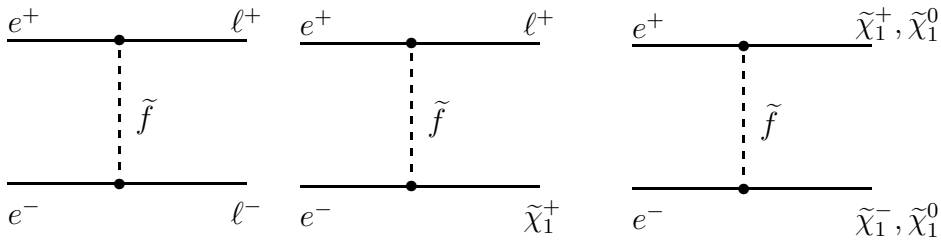
Signals for  $R$ -parity violation at colliders can, therefore, be classified into two kinds:

- where the magnitude of the coupling is important, such as, *e.g.*, virtual sparticle exchanges and single sparticle production; and
- where the magnitude is irrelevant, *e.g.* in LSP decay.

Since, at the present time, there exist no experimental signs of physics beyond the SM, what one can obtain from an analysis of the first kind of signal are bounds on the  $R$ -parity-violating couplings modulo assumptions about the other parameters of the MSSM. On the other hand, the second kind of coupling leads to bounds on the MSSM parameter space *in the presence of*  $R$ -parity violation, without giving much information on the magnitude of the coupling involved. Most current studies at colliders have been of the latter type; studies of the former kind, though not entirely neglected, were really triggered-off by the observation of a possible leptoquark signal at HERA in 1997 (see 3.3 below). However, similar studies are possible at almost all types of colliders, and some of these will be discussed below.

### 3.1 $R$ -Parity Violation at LEP

At the CERN Large Electron Positron (LEP) collider three kinds of signals are possible. Typical diagrams for these are shown in Figure 3, though the list is far from exhaustive. The first two involve direct values of the couplings, while the last involves decay of the LSP, which is also an end-product of decays of the chargino.



**Figure 3.** Typical Feynman diagrams leading to signals for  $R$ -parity violation at LEP.

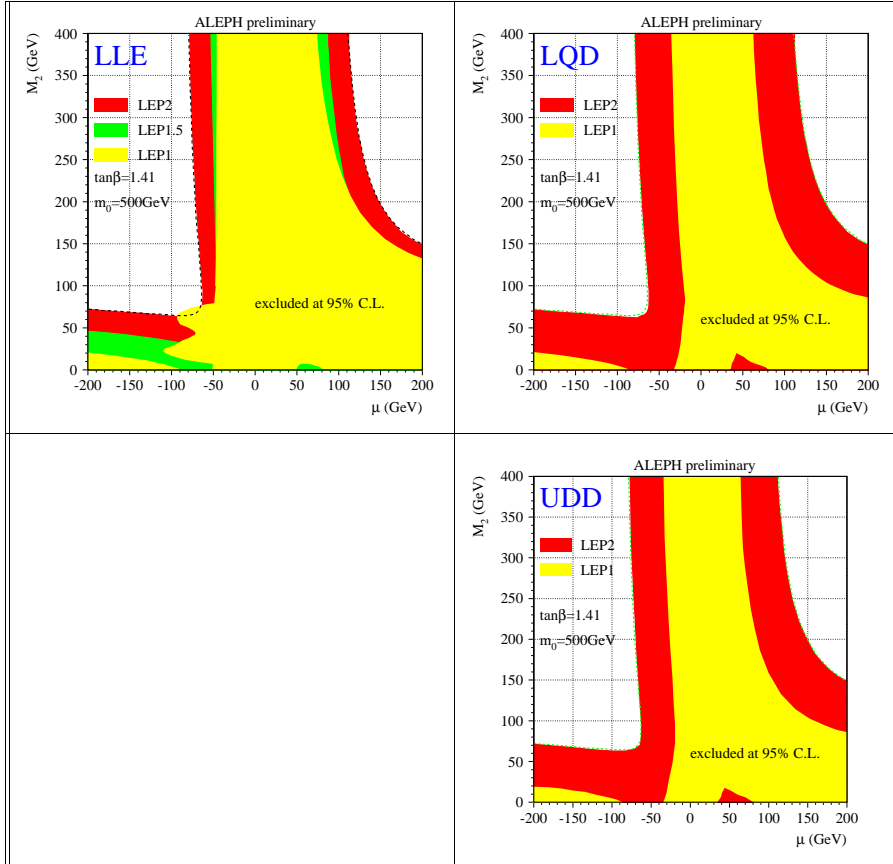
The first diagram on the left shows the effect of a virtual sfermion exchange on dilepton (diquark) production in  $e^+e^-$  collisions. Sneutrino exchanges can lead to the production of dileptons in the final state, if the  $R$ -parity-violating coupling is of the  $\lambda$  type, while squark exchanges can lead to production of a dijet final state, if the  $R$ -parity-violating coupling is of the  $\lambda'$  type. The cross-section varies as the fourth power of the relevant coupling and hence, is rather sensitive to its value. Of course, identification of the coupling will depend on the tagging efficiency, which is high for leptons, reasonable for  $b$ -quarks, small for  $c$ -quarks and non-existent for lighter quarks.  $t\bar{t}$ -production is, of course, forbidden by kinematics. An analysis shows[25] that once the full data are available from LEP-2, one can obtain modest bounds on the parameter space formed by the  $R$ -parity-violating coupling and the mass of the exchanged sparticle.

The second diagram (in the middle) shows one possibility for single sparticle production at LEP-2 through  $R$ -parity-violating couplings. Originally discussed in Ref.[26], a detailed study has been performed recently in Ref.[27]. Other possibilities involve final



states with a neutrino and a neutralino. In this case, one has to take into account the final states formed by chargino or neutralino decay — ultimately LSP decay — in identifying signals. These amplitudes are linear in the  $R$ -parity-violating coupling involved, which means that while they are less sensitive to the magnitude of  $\lambda$ , they are sensitive to possible phases in the  $R$ -parity-violating couplings[28]. The analysis of Ref.[27] shows, in general, that LEP-2 cannot really significantly improve the bounds already existing on  $R$ -parity-violating couplings. It hardly needs to be mentioned that this is just a limitation of the energy and luminosity available at LEP-2: we can get far better results at a 500 GeV or 1 TeV machine with higher luminosity.

The third diagram on the right of Fig. 3 actually encompasses a large number of possibilities. One can produce any one of the pairs  $\tilde{\chi}_1^+ \tilde{\chi}_1^-$  or  $\tilde{\chi}_1^0 \tilde{\chi}_1^0$  or  $\tilde{\chi}_1^0 \tilde{\chi}_2^0$  depending on the point in the MSSM parameter space where the analysis is made. The heavier chargino and neutralino states are usually forbidden by kinematics. All these states will decay into a pair of LSP's  $\tilde{\chi}_1^0 \tilde{\chi}_1^0$  which can then decay through the  $R$ -parity-violating couplings listed above[29]. The large multitude of possible final states has been extensively studied by the four LEP collaborations[30] and has been used to constrain the MSSM parameter space. Some of these results, from the ALEPH Collaboration, are shown in Figure 4.



**Figure 4.** ALEPH bounds on the  $(M_2, \mu)$  plane for  $\tan \beta \simeq \sqrt{2}$ , assuming the presence of different  $R$ -parity-violating couplings. The  $\tilde{\chi}_1^0$  is assumed to be the LSP.

For this study, sfermion masses were evolved from a common  $m_0 = 500$  GeV which made them too heavy to affect the decay modes of any gaugino (other than the LSP).

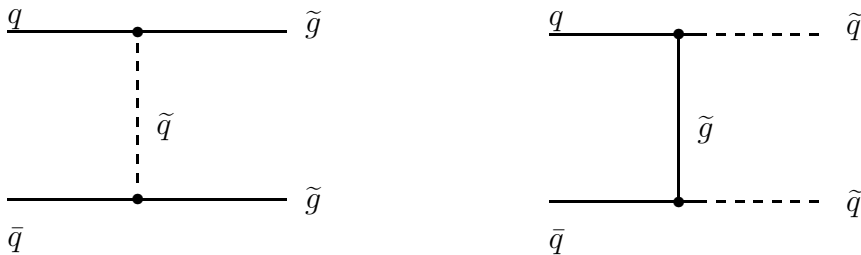
Moreover, the value of  $\tan\beta$  is taken to be 1.41 ( $\simeq \sqrt{2}$ ), as indeed it is for several other studies made by the LEP collaborations. This is about the lowest value allowed by current LEP constraints on the plane formed by  $\tan\beta$  and the mass of the lightest Higgs boson  $h^0$ . Larger values of  $\tan\beta$  tend to exclude greater regions of the parameter space, hence, the shaded regions in Fig. 4 may be taken as conservative bounds.

An important consequence of these constraints on the parameters of the MSSM is that they can be translated into bounds on the mass of the LSP, here assumed to be the  $\tilde{\chi}_1^0$ . This is because it is precisely these parameters which go into the construction of the neutralino mass matrix. The DELPHI Collaboration has, for example, presented results from data collected at 183 GeV, showing that  $M_{\tilde{\chi}_1^0} > 45$  GeV (unless  $\tan\beta \simeq 1$ ), and having an absolute lower bound of 27 GeV. For a 200 GeV run, these numbers are expected to increase to 50-55 GeV and 35 GeV respectively.

A few more remarks may be appropriate before closing our discussion on LEP. As with all other areas where new physics has been predicted, LEP has turned up negative results<sup>4</sup> and hence provided a set of excellent (but disappointing!) bounds on the MSSM scenario where  $R$ -parity is not conserved. Unless this version of supersymmetry is just around the corner, waiting to be discovered, the best we can expect from the future run(s) of LEP-2 is a marginal improvement in the bounds discussed above.

### 3.2 $R$ -Parity Violation at the Tevatron

At the Tevatron — as indeed at any hadron collider — the second kind of signal from  $R$ -parity violation can be further divided into two classes: those arising from electroweak production of sparticles, and those arising from QCD production of strongly-interacting sparticles, *i.e.* squarks and gluinos. For the first kind, the diagrams are analogous to those of Fig. 3, with the  $e^+e^-$  pair being replaced by a pair of light quarks. Typical diagrams giving rise to the production of squarks and gluinos are shown in Fig. 5.



**Figure 5.** Typical Feynman diagrams leading to squark and gluino production in  $q\bar{q}$  annihilation at a hadron collider. One can also draw similar diagrams with initial-state gluons.

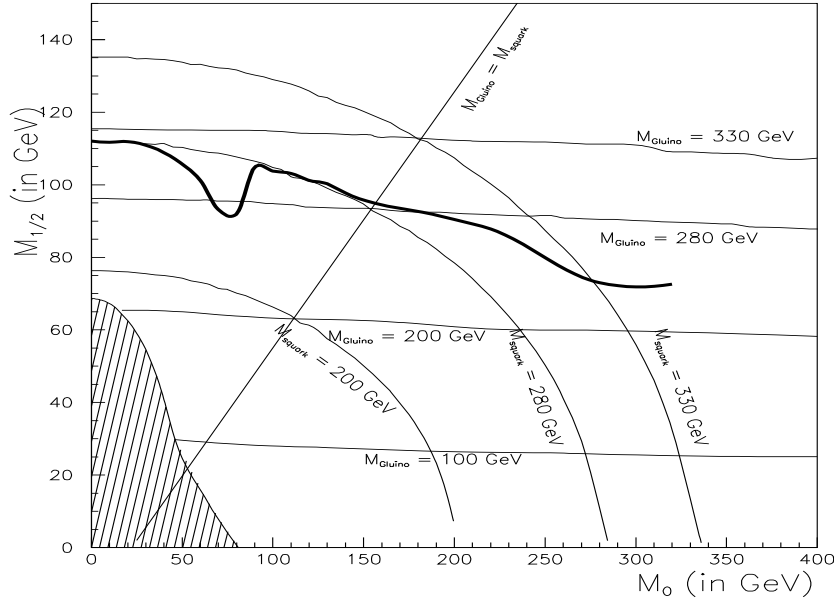
Let us first consider the case of electroweak production. The diagram analogous to the one on the extreme left of Fig. 4 can lead, for example, to dilepton production through squark exchange. This could lead to changes in the observed Drell-Yan cross-sections at the Tevatron. As the observed cross-sections show remarkable agreement with the SM

<sup>4</sup>A few years back, it was thought that ALEPH had observed an excess of events in the four-jet channel, from their 130–136 GeV data, which appeared to come from the production of a pair of non-SM particles. While it was shown[31] that this could not come from charginos or neutralinos decaying through  $\lambda''$  couplings, a strong case was made for a pair of light squarks[32] or sleptons[22] decaying through  $R$ -parity-violating couplings. However, a repeat run of LEP at 130-136 GeV failed to show up any further excess and hence the four-jet ‘anomaly’ was consigned to the dustbin of history.

(*déjà vu*) one can obtain interesting bounds[33] on the parameter space from these. It is also possible to have contributions to final states where a pair of quarks is produced from such diagrams (with slepton exchange). The best measured of these cross-sections is the  $t\bar{t}$  cross-section and it can be used to constrain the parameter space[34].

The diagrams analogous to the middle one in Fig. 3 can lead to single sparticle production at the Tevatron. Typically in the presence of  $\lambda'$  couplings we could expect a final state with a hard lepton and a chargino or neutralino which would then decay in the manner described above to an LSP and then to multi-fermion states. Analogous processes with jets could also occur. An investigation of these possibilities is desirable, especially in the context of Run II of the Tevatron, where reasonably small values of the couplings could be probed.

The diagram on the extreme right of Fig. 3 will have analogues at the Tevatron where not only a pair of charginos or a pair of neutralinos are produced, but a chargino can be produced in association with a neutralino. These, of course, are what lead to the well-known trilepton signatures. If  $R$ -parity is violated, the decay of the LSP will lead to various multi-fermion final states. For example, if there are  $\lambda$ -type couplings, one typically predicts signals with upto 7 leptons in the final state. A detailed investigation of these is awaited.



**Figure 6.**  $D0$  bounds on the MSSM parameter space assuming the presence of  $R$ -parity-violating  $\lambda'$  couplings. An  $mSUGRA$  (or  $cMSSM$ ) framework is assumed.

Of greater interest than the above signals, however, are those which arise from QCD production of squarks and gluinos, which (naturally) have far larger cross-sections. The squarks and gluinos will decay[35] through  $R$ -parity conserving modes (cascades), with a pair (at least) of LSP's in the final state. When these decay, we again have a multitude of possible final states, which have to be looked for individually. Once again, searches for these have been performed, but have yielded negative results so far, leading to bounds on the MSSM parameter space. Figure 6 shows the recent bounds made public by the  $D0$  Collaboration[36]. Clearly, these lead to bounds on the squark and gluino masses over 200 GeV each and thus complement the negative results of leptoquark searches[37]

at the Tevatron. One can expect considerable improvement in these bounds in Run II which essentially push the mass limits from  $\sim 200$  GeV to  $\sim 300$  GeV[38].

Of particular interest at the Tevatron are like-sign dilepton signals from  $\tilde{\chi}_1^0$  decay in the case of  $\lambda'$  couplings, since they constitute ‘smoking gun’ signals for Majorana fermions, which are present in the MSSM, but not in the SM. These have been experimentally investigated[39] in the context of the HERA anomaly (section 3.3) using a model described in Ref.[40]. However, a more general analysis has been advocated[41] (with justification) and should be performed.

Other processes of interest at the Tevatron are the possibility of slepton resonances[42] and the possibility of single top quark[43] and squark[44] production. Though current bounds on the couplings from these processes are not very impressive, considerable improvement can be expected from Run-II data.

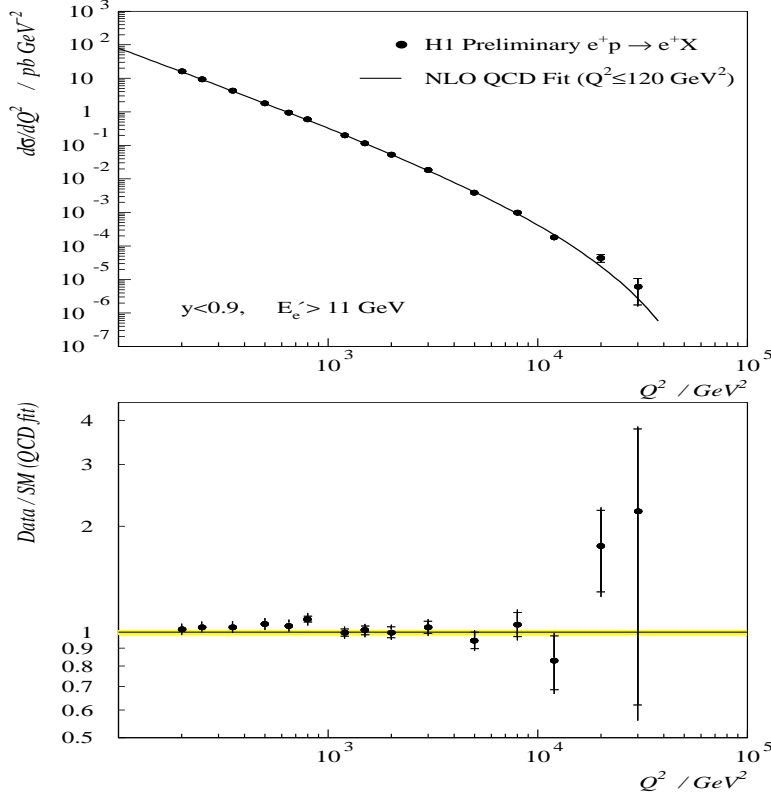
The LEP-2 bounds should be taken into account in studies at the Tevatron and also whenever future studies of  $R$ -parity violation are carried out. One difficulty in the way of this is the slight difference in philosophy between the four LEP collaborations and the CDF and D0 collaborations at Tevatron as to what constitutes the  $c$ MSSM, which underlies all these studies. The CDF and D0 collaborations use the most stringent form of the  $c$ MSSM, where the only free parameters are (as explained above)  $m_0$ ,  $M_{1/2}$ ,  $A$ ,  $\tan\beta$  and the sign of the Higgsino mixing parameter  $\mu$ . The magnitude of  $\mu$  is fixed by the condition that the electroweak symmetry be broken (by a Coleman-Weinberg-type mechanism) at the right scale. The LEP collaborations, however, treat the magnitude of  $\mu$  as a free parameter, which essentially means that the gluino mass  $M_3$  is not unified with the other gaugino masses at the GUT scale. Of course, the gluino mass has no direct role to play at LEP, but a comparison of LEP and Tevatron results would be possible only if both are presented within the same set of assumptions[45].

The reverse side of the same coin is the question as to how much these bounds can be relaxed if one does not make the assumptions which go into the  $c$ MSSM. Of course, it must be admitted that in this case there are so many free parameters that a phenomenological analysis becomes rather bewildering. Nevertheless, it would be interesting to know if there exist some absolute bounds from the Tevatron and LEP-2 data on the sparticle masses. However, no such detailed analysis exists at the present point of time.

### 3.3 $R$ -Parity Violation at HERA

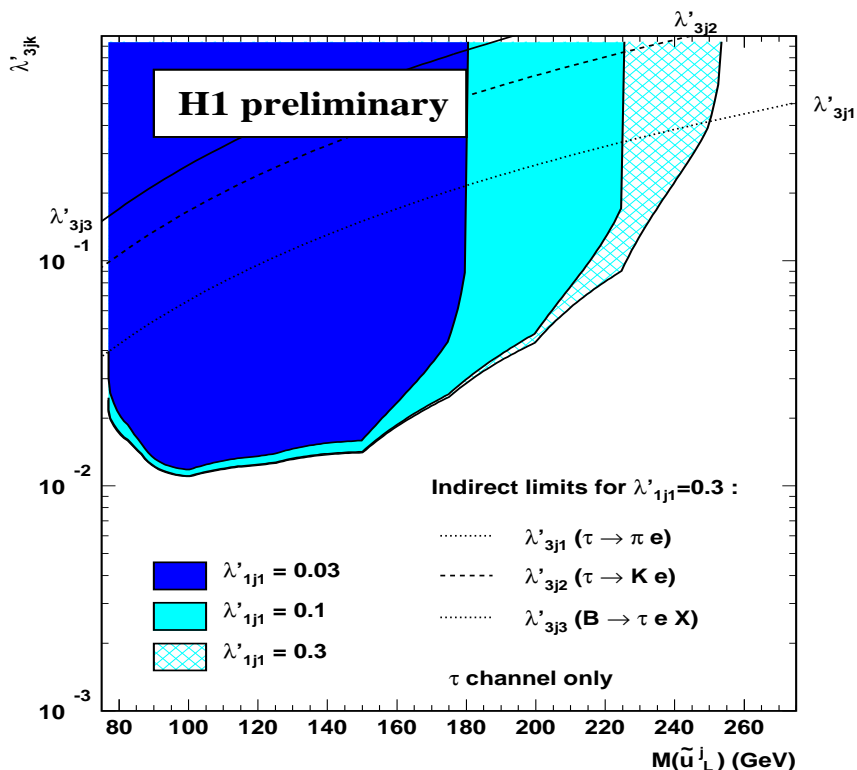
Since HERA is an  $ep$  collider, it is the ideal machine to look for first-generation leptoquarks. This was pointed out long ago[46, 47]. However, intense interest in this subject was generated only in 1997, when the H1 and ZEUS Collaborations simultaneously reported[48] that their data showed an ‘excess’ of back-scattered (high- $Q^2$ ) positrons seen in  $e^+p$  collisions. The signals seemed to show the classic features of resonance production of an intermediate particle which was quickly identified with either a leptoquark or a squark with  $\lambda'_{1j1}$  or  $\lambda'_{132}$  couplings[23]. There was considerable initial excitement, despite warnings[50] that the H1 and ZEUS data were far from consistent with each other. However, this euphoria was soon dissipated. In the first place, leptoquark searches at the Tevatron seemed to rule out a squark or leptoquark with a mass of around 200 GeV, which was required to explain the HERA anomaly. Moreover, further data taken by the H1 and ZEUS collaborations failed to produce any further ‘excess’ events in the high- $Q^2$

region. Though the present data still shows some departure from the SM, as shown in Figure 8, the consensus appears to be that the ‘excess’ events were due to statistical fluctuations and do not constitute a sign for new physics beyond the SM. In fact, both H1 and ZEUS Collaborations have now published[51] bounds on the MSSM parameter space analogous to those at the LEP and the Tevatron. In the final analysis, the only positive result of the ‘excess’ has been to bring to common notice the fact that  $R$ -parity violation is a natural SUSY scenario and not an exotic one.



**Figure 7.** H1 data on the  $Q^2$  distribution of neutral current events. The upper graph shows the actual distribution, while the lower one shows the data scaled to the SM NLO prediction (the solid line in the upper graph). The small excess at high values of  $Q^2$  is clearly within the  $2\sigma$  errors.

Several signal modes are actually possible[47] at HERA, when we include the possibility that  $R$ -parity violation is weak and appears only through decays of the LSP. Some of these involve resonance production of a squark, as suggested in the case of the high- $Q^2$  anomaly, but there are other possibilities, such as associated production of a squark or slepton with a chargino or neutralino, followed by  $R$ -parity-violating decays of both of these. The somewhat complicated signals for these have been analyzed in Ref.[47] and experimental searches have been made by the H1 Collaboration[49]. Some of the (negative) results go into the current H1 and ZEUS bounds. An example of these is shown in Fig. 8, which illustrates H1 bounds on some of the  $R$ -parity-violating couplings as a function of the mass of the exchanged squark. This is, in a sense, paralleled by similar graphs produced by the LEP Collaborations. As HERA collects more data, one can expect significant improvement in these bounds, both in those which bound the  $M_2$ - $\mu$  plane, and in those which constrain the  $\lambda'$ - $M_{\tilde{q}}$  plane. Probably we can also expect the  $Q^2$  distribution to approximate the SM prediction better and better as time goes by.



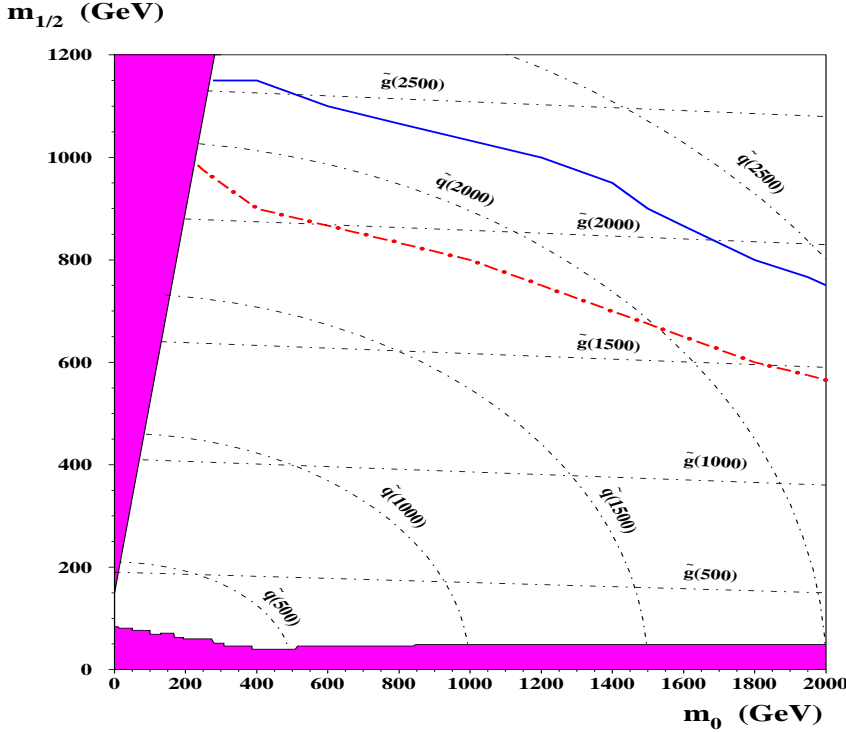
**Figure 8.** *H1* bounds on the  $R$ -parity violating coupling  $\lambda'_{3jk}$  as a function of the mass of the exchanged sparticle, assuming different values for  $\lambda'_{1j1}$ .

One interesting aspect of HERA physics, which has not attracted much attention, is the excess of events with multiple leptons, especially muons, in the final state. There seems to be little explanation for these in the SM or in most beyond-SM scenarios, except as unwanted fluctuations. While this may well turn out to be the true explanation, it is also possible to think[52] of a squark resonance being produced through a  $\lambda'$  coupling and decaying indirectly through a  $\lambda$  coupling into multiple leptons in the final state. Such a scenario would be of great interest should the multi-lepton excess at HERA persist, but at the moment it is premature to think of it as a signal for  $R$ -parity-violation.

### 3.4 $R$ -Parity Violation at Future Colliders

Except for the Tevatron (Run-II), none of the other high energy colliders seem to promise significant improvements in discovery limits for  $R$ -parity-violating couplings in the near future. We must, therefore, look to the colliders of the future, which are expected to have much higher energy and luminosity, if significant improvements are to be expected. The most important of these machines is the LHC, which is scheduled to begin operation in the year 2005. Searches for  $R$ -parity violation at the LHC follow the paradigm set for the Tevatron, the main difference being the higher energy and luminosity. Though no *detailed* study of  $R$ -parity violating signals at the LHC exists as yet, it is conceivable that some signals which are too small to be observable at the Tevatron might become observable at the LHC. The reverse side of the coin is that backgrounds — especially QCD backgrounds — could also be large. However, it is possible to be upbeat about  $R$ -parity violation searches at the LHC in view of the

fact that preliminary studies carried out by the CMS and ATLAS collaborations at the LHC[53] promise to carry the squark and gluino mass search limits, in the presence of  $R$ -parity-violating couplings, to the vicinity of 2 TeV each. This represents an increase of nearly an order of magnitude over the present bounds from the Tevatron.



**Figure 9.**  $5\sigma$  discovery potential of CMS in  $mSUGRA$  for  $\mu < 0$ ,  $\tan\beta = 2$  and  $A_0 = 0$ . In the region below the solid (blue) curve a signal of  $R$ -parity-violating supersymmetry via  $\lambda_{121} = 0.05$  would be discovered ( $5\sigma$  for an integrated luminosity of  $10^4 \text{ pb}^{-1}$ ). The dashed (red) curve corresponds to the discovery potential for a signal via  $\lambda_{233} = 0.06$ . In the shaded (pink) region,  $mSUGRA$  is not valid or  $\tilde{\chi}_1^0$  is no longer the LSP.

In Fig. 9, we illustrate the reach of LHC in the above parameters using a plot from a preliminary study carried out by the CMS Collaboration[53]. In this plot contours of the squark and gluino masses are shown in the  $m_0$ - $m_{1/2}$  plane. The (pink) shaded regions show the regions where this analysis is not valid, either because there exists no phenomenologically viable solution to the renormalization group equations at the electroweak scale, or because the lightest neutralino is no longer the LSP. The latter case will have its own distinctive signatures — the fact that the relevant region is shaded merely represents the fact that these are not covered by the analysis that generated this plot. It is immediately apparent that for an optimistic  $\lambda_{121} = 0.05$ , where the LSP decays into multiple electron and muon states, one can essentially probe all values of  $m_{1/2}$  upto about 800 GeV, which corresponds to a gluino mass of nearly 2 TeV. The situation is worse for the  $\lambda_{233}$  coupling, despite its higher value, essentially because of the difficulties of identifying  $\tau$ -leptons from the enormous backgrounds at the LHC.

The ATLAS Collaboration has made a very similar study, basing their signals on a  $\lambda$ -type coupling, where the final state contains multi-leptons and hence is easier to detect. However, it is not clear how efficient the LHC would be as a detector of  $R$ -parity violation if the couplings are of  $\lambda'$  or  $\lambda''$  type. All that can be said with certainty is

that detection of the resultant signals would be complicated by the presence of large QCD backgrounds and detection of like-sign dileptons might be the best bet[41] for such studies. A detailed analysis is awaited.

Next to the LHC, which is certainly going to be built, we need to consider the Next Linear Collider (NLC), which is the usual acronym for a 500 GeV  $e^+e^-$  collider (necessarily of the linac type). Various possibilities for the NLC are under serious consideration. It is certainly going to be run in the  $e^+e^-$  collision mode, but other alternatives are an  $e^-e^-$  collision mode, and, using laser back-scattering techniques,  $e\gamma$  and  $\gamma\gamma$  collision modes as well. Detailed studies of  $R$ -parity violation at the NLC have not really been carried out. Preliminary studies show[26, 54] that some order can probably be extracted out of a multitude of signals in the  $e^+e^-$  mode — in fact, for  $\lambda$ -type couplings, we might have rather spectacular multi-lepton signals with practically no background. For  $\lambda'$  and  $\lambda''$  couplings, it might be possible[54] to reconstruct the mass of the decaying LSP, assumed to be the  $\tilde{\chi}_1^0$ . Similar studies could also be carried out in the  $e\gamma$  mode[55]. Except for some initial studies[56], the  $e^-e^-$  mode has not been looked into in any detail, though this might be thought of as an ideal machine to study lepton number violation. Except for a preliminary study in Ref. [57], the  $\gamma\gamma$  mode has not been considered as yet for  $R$ -parity-violating signals. A study of this is probably worth carrying out.

Another possibility, which is now receiving serious consideration, is that of a muon collider. Muons have an advantage over electrons in their heavier mass, which inhibits synchrotron radiation. It might, therefore, be possible to build a  $\mu^+\mu^-$  collider with much higher energy and luminosity than even the NLC. One important advantage of such a machine would be in its great potential for studying flavour physics, of which  $R$ -parity violation provides one example. Some preliminary studies of  $R$ -parity violation at a muon collider have been carried out[58] and they seem to yield promising results.

It might also be possible to have a muon-proton collider. At Fermilab and LEP, for example, high energy proton beams are already available, and it would only be necessary to bring these into collision with the muon beam. Such a machine would be the second generation equivalent of HERA and could probe, among other things,  $R$ -parity-violation through  $\lambda'$  couplings to great accuracy[59].

A muon collider may also be thought of as a copious source of neutrinos which come from the decay of muons in the high-luminosity beams. While it has been speculated that such neutrino beams can be used for long-baseline experiments to test neutrino masses and mixings, it might also be possible, by allowing the high energy neutrino beams to impinge on a target, to derive bounds on  $R$ -parity-violating couplings. Though similar studies have been done for ultra-energetic neutrinos from cosmic sources[60], the question of neutrinos from a muon collider has not yet been addressed.

#### 4. SUMMARY AND OUTLOOK

$R$ -parity conservation, as we have discussed, is a pleasing but not an absolutely necessary feature of the MSSM. Of course, any arguments for or against such a symmetry in the MSSM are essentially speculative, since, in the first place, we have no empirical evidence that SUSY exists — whether in the  $R$ -parity-conserving or  $R$ -parity-violating form. The question of  $R$ -parity should therefore be approached with an open mind. What is known for certain is that there are useful bounds from low-energy studies and from collider experiments which are still running or will run again in the near future. Of



these, bounds from LEP and the Tevatron are essentially indirect bounds, in that they arise from a search for LSP decay modes. More direct bounds come from HERA, which, after generating some initial excitement, is now expected to provide steadily improving bounds, as more and more data is accumulated. Major improvements may be expected from Run-II of the Tevatron, though a hadron machine has intrinsic problems when dealing with multi-jet final state signals. It is, therefore, necessary to continue investigations of possible signals for  $R$ -parity violation not only at the present generation of colliders, but also at the machines of the future, of which the Large Hadron Collider (LHC) at CERN is a certainty. Less certain, but quite probable, are linear  $e^+e^-$  colliders while muon colliders are now being seriously considered. A major part of the physics agenda at such machines would be SUSY searches and hence, studies of  $R$ -parity violation are essential to have a comprehensive picture.

It must be noted that present work in  $R$ -parity violation has merely touched the tip of the iceberg, since it has usually been carried out within a very specific set of assumptions — such as, *e.g.* a neutralino LSP. Apart from studies connected with experimental searches, most studies at future colliders have been rather sketchy in addressing issues such as, for example, QCD corrections and backgrounds, both from the SM as well as from the  $R$ -parity-conserving sector of the MSSM. In fact, till now, studies of  $R$ -parity violation have usually assumed that the rest of the MSSM does not matter, an assumption which can, with justification, be challenged. The future will probably see a number of the issues discussed here being addressed and, with a bit of luck, the world may turn out to be supersymmetric and  $R$ -parity-violating as well.

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